**Limited Arbitrage and Uniqueness of Equilibrium in Strictly Regular Economies**

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July 1996

Discussion Paper Series No. 9596-21

dp 9596-21
pages: 10

# Limited Arbitrage and Uniqueness of Equilibrium in Strictly Regular Economies

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**Abstract**

In strictly regular economies limited arbitrage is sufficient for the global invertibility of demand, and necessary and sufficient for the uniqueness of equilibrium. This result is established using algebraic topology and holds in economies with short sales, and with finitely or infinitely many markets.

## 1 Introduction

Limited arbitrage is a unifying condition for resource allocation. Defined on the endowments and the preferences of a market's traders, it is necessary as well as sufficient for the existence of a competitive equilibrium, for a nonempty core and for the existence of desirable social choice rules.[1] This paper analyzes the connection between limited arbitrage and unique market equilibrium. I study a class of *strictly regular economies*, which are characterized by strictly convex preferences[2] and a demand with a nonvanishing Jacobian on its domain. Strictly regularity is a stringent condition, but it is simple and easy to verify. Within this class of economies limited arbitrage is necessary and sufficient for uniqueness of market equilibrium.

The results presented here encompass economies with short sales and with infinitely many markets, which were neglected in the literature on uniqueness of equilibrium. The excess demand function need not be well defined at all prices.

How does limited arbitrage work? As already pointed out, limited arbitrage is necessary for existence of equilibrium. The necessity part of the results presented here is therefore clear. The sufficiency derives from a new result, an application of [10]. I show that in strictly regular economies with limited arbitrage the excess demand function is a proper map with a nonempty and contractible image. From this, using algebraic topology, I show that the excess demand is globally invertible and, in particular, there exists a unique equilibrium.

The conditions presented here are binding. If the economy is not strictly regular, then limited arbitrage does not imply unique equilibrium.[3] Strict regularity does not by

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[1] See Chichilnisky [3], [4], [5][12], [6], [7], [8], [9], Chichilnisky and Heal [?], [12], [13].

[2] The results also hold for preferences whose indifferences contain no half lines, which are more general than strictly convex preferences.

[3] Consider for example a two trader economy where both trades have identical linear preferences. This economy satisfies limited arbitrage but it is not strictly regular because preferences are not strictly convex, and it has infinitely many equilibria allocations.

itself ensure uniqueness unless the excess demand is a proper map with a nonempty and contractible image, conditions which are derived here from limited arbitrage.

The paper is organized as follows. Section 2 contains definitions, Section 3 the main results and Section 4 discusses related literature. An Appendix provides the topological background.

## 2 Definitions

An market $\mathsf{E} = \{X, \Omega_h, u_h, h = 1, ..., H\}$ has $H \geq 2$ traders, and trading space $X = R^{N+1}$, with $N \geq 1$, or $X = l_2$,[4] the space of square summable sequences of real numbers with a finite measure. Trader $h$ has a preference represented by a continuous, strictly concave increasing function $u_h : X \to R$.[5] For trader $h$ define the *global cone* of directions where utility never ceases to increase, introduced in [7], [4] and [3]:

$$G_h(\Omega_h) = \{x \in X \text{ and } \sim \exists \max_{\mu \geq 0} u_h(\Omega_h + \mu x)\}, \tag{1}$$

and its *market cone*

$$D_h(\Omega_h) = \{z \in X : \forall y \in G_h(\Omega_h), \langle z, y \rangle > 0\} \tag{2}$$

$D_h(\Omega_h)$ is the convex cone of prices assigning strictly positive value to all directions in $G_h(\Omega_h)$. Both cones are the same for all $\Omega_h \in X$ under the assumptions; when preferences are strictly convex in $R^N$, the cone $D_h$ is open and convex for each $h$, see [3], [7], [8], and in $l_2$ we assume that $D_h$ is open and convex as well.[6] Good 1 is the *numeraire*; to ensure its desirability I assume:

**Desirability condition:** for all $h$, the vector $(1, 0, ..., 0, ...)$ is in $G^o_h(\Omega_h)$.[7]

**Definition 1** $\mathsf{E}$ *satisfies* ***limited arbitrage*** *when*

$$(LA) \ D \equiv \bigcap_{h=1}^{H} D_h \neq \emptyset.$$

By construction, the excess demand function $Z_h$ of each trader $h$ is defined only on the set $D$; I assume these functions are smooth on $D$, a condition which is trivially satisfied if $D$ is empty. Observe that the desirability condition implies that $\forall p \in D$, its first coordinate $p_1 \neq 0$. Since by Walras Law $\forall p \in D, \langle p, Z(p) \rangle = 0$, to identify a market equilibrium it suffices to find a zero of the composition map $\pi \circ (\sum_{h=1}^{H} Z_h) : D \to R^{N-1}$, where $\pi : R^N \to R^{N-1}$ is the projection map on the $N-1$ coordinates other than 1. Similarly in the infinite dimensional case $\pi \circ (\sum_{h=1}^{H} Z_h) : D \to l_2$, where $\pi : l_2 \to l_2$.[8] This composition map is called from now on the *excess demand* of the economy, and is denoted $Z : D \to X$. A **competitive equilibrium** $p^*$ is a zero of the excess demand

[4] $l_2 = \{(x_n)_{n=1,...} : \sum x_n \lambda^n < \infty$, for a given $\lambda, 0 \leq \lambda \leq 1\}$.

[5] Increasing means that $u(x) \geq u(y)$ if $x \geq y$, and $u(x) > u(y)$ if $x >> y$. Preferences need not be strictly increasing coordinatewise. If $x, y \in R^N$, $x \geq y \Leftrightarrow \forall i \ x_i \geq y_i$, $x > y \Leftrightarrow x \geq y$ and for some $i, x_i > y_i$, and $x >> y \Leftrightarrow \forall i, \ x_i > y_i$.

[6] Since the positive orthant in $l_2$ has empty interior, this implies that not all vectors in $D_h$ are positive. Alternatively one can require the cone condition introduced by Chichilnisky and Kalman in 1980 [14] and renamed properness later, see also Le Van [17].

[7] $G^o_h(\Omega_h)$ is the interior of $G_h(\Omega_h)$.

[8] As is well known, when the vector $Z(p)$ has all but one coordinate equal to zero, then $Z(p)$ *is* the zero vector, because $\forall p \in \Delta, \langle p, Z(p) \rangle = 0$.

function i.e. $p^* \in Z^{-1}(0)$. When preferences are strictly convex, for each equilibrium price there is only one corresponding equilibrium allocation.

Let $M$ and $N$ be two manifolds of the same dimension.[9] A map $f : M \to N$ is *globally invertible* when it is one to one and $f(M) = N$. Given topological spaces $X$ and $Y$, a continuous map $f : X \to Y$ is called *proper* when the inverse image of every compact set $C$, $f^{-1}(C)$, is compact. The unit sphere is $S = \{p \in R^N : \sum_{i=1}^{N+1} p_i^2 = 1\}$. When $X = l_2$ then $S = \{p \in l_2 : \sum_{i=1}^{\infty} p_i^2 \lambda^i = 1$. The intersection of the set $D$ with $S$ is homeomorphic to[10] an open bounded ball in $R^N$ or $l_2$ respectively. It suffices to work on this set $D \cap S$ of prices, which we do from now on. For simplicity we use the same notation $D$ for this.

**Definition 2** *When $X = R^N$ the market $E$ is strictly regular when (a) it has strictly convex preferences and (b) the excess demand $Z : D \to X$ satisfies the desirability condition and has a nonvanishing Jacobian.*

The following defines strict regularity in infinite economies. For the definition of Frechet derivative see [2].

**Definition 3** *When $X = l_2$ the market $E$ is strictly regular when (a) it has strictly convex preferences and (b) the excess demand $Z : D \to X$ satisfies the desirability condition and its Frechet derivative $DZ$ is invertible with its norm bounded below.*[11]

# 3 Market Equilibrium

The following result applies to finite or infinite economies:

**Theorem 1** *Under limited arbitrage, the excess demand function $Z : D \to X$ of a strictly regular economy is a proper map which it is globally invertible.*

**Proof.** Observe that if $p^j \to (1, 0..., 0, ...)$ then for trader $h$ all the coordinates of the trader's excess demand, except the first, may are bounded. However, if $p^j \to p \in \partial D_h$ and $p \neq (1, 0, ..., 0, ...)$ by the definition of $D_h$ and by the monotonicity and strict convexity of the utilities some coordinate of trader $h's$ demand other than the first must increase without bound. The desirability condition ensures that $(1, 0, .., 0) \notin \partial D_h$. Furthermore $p^j \to p \in \partial D$ implies $p^j \to p \in \partial D_h$ for some $h \in \{1, ..., H\}$. Therefore $p \in \partial D \Rightarrow p \neq (1, 0, ...)$. Therefore as shown above since preferences are monotonic and strictly convex, and $p \in \partial D_h$ for some $h$, then $p^j \to p \in \partial D \Rightarrow p \neq (1, 0, ...) \Rightarrow lim_{j \to \infty} \parallel Z(p^j) \parallel = \infty$.

The next step is to show that when $X = R^N$ the fact that $p^j \to p \in \partial D \Rightarrow \parallel Z(p^j) \parallel = \infty$ implies that the excess demand is proper. In the case $X = l_2$, I will also use the fact that the Frechet derivative is bounded above zero. Therefore in all cases the excess demand $Z$ of a strictly regular economy is proper.

First consider the finite dimensional case, $Z : D \to R^N$. We need to show that the inverse image of any compact set $C$ in $R^N$, $Z^{-1}(C)$, is compact. The proof is by contradiction. If $Z^{-1}(C)$ was not compact in $D$ then there would exist a sequence of points $(p^j)_{j=1,2,...} \subset Z^{-1}(C)$ containing no convergent subsequence in $Z^{-1}(C)$. Since $D \subset \Delta$, $D$ is a bounded subset and since $X = R^N$ it is precompact. Therefore this can

[9] Finite or infinite. In the latter case the manifold must be a Banach manifold, for otherwise the inverse function theorem may not hold. Unless otherwise specified all manifolds are smooth i.e. $C^k$, with $k \geq 2$ connected and without boundary, and all maps are smooth.

[10] I.e. in a one-to-one onto bicontinuous correspondence with an open bounded ball in $R^N$ or $l_2$ respectively.

[11] I. e. $\exists \varepsilon > 0 : \forall p \in D, \|DZ(p)\| > \varepsilon$. Observe that an economy with strictly convex preferences is always strictly regular when $D$ is empty.

only happen when $p^j \longrightarrow p \in \partial D$. But in this case, by strict regularity the image is not bounded, contradicting the assumption that $C$ is compact. This completes the proof for the finite dimensional case.

Next consider the infinite dimensional case. We need to show that the inverse image of any compact set $C$ in $l_2$, $Z^{-1}(C)$, is compact. This set is always closed because $Z$ is continuous. If $Z^{-1}(C)$ was not compact in $D$ then there would exist a sequence of points $(p^j)_{j=1,2,...} \subset Z^{-1}(C)$ containing no subsequence with a limit in $Z^{-1}(C)$. Consider now the image of this sequence, $Z(p^j)_{j=1,2,...}$. By the compactness of $C$ there is a subsequence, denoted also $Z(p^j)$, which has a limiting point $y \in C$. By assumption any sequence in its inverse image $z^j \in Z^{-1}(Z(p^j)$ does not converge. This may occur when $z^j \longrightarrow \partial D$; however as seen in the first part of this proof, by the hypothesis of this theorem we would have a contradiction with the assumption that $C$ is compact. Therefore $z^j$ does not converge to $\partial D$. In this infinite dimensional case another possibility arises: the sequence $z^j$ may not converge because although $Z^{-1}(C)$ is closed and bounded, it is not compact. Indeed, closed bounded sets in $l_2$ are generally not compact.

However, the sequence $z^j$ is bounded and contained in a closed set. Therefore it can only fail to have a limit when $z^j$ is not Cauchy:[12] if it is a Cauchy sequence, by the completeness of $l_2$, it would have a limit because $Z$ is continuous so that $Z^{-1}(Z(p^j))$ is closed. Therefore $z^j$ is not Cauchy, i.e. $\exists \varepsilon > 0 : \forall n, m > N \; \|z^n - z^m\| > \varepsilon$. However by the assumption on $DZ$, this implies that $Z(p^j)$ is not Cauchy either, which contradicts the fact that $Z(p^j) \longrightarrow p$. Since the contradiction emerges from assuming that $Z$ is not proper, $Z$ must be proper. I have therefore shown that in all cases, $Z$ is a proper map. Observe that when $D$ is empty, then the map $Z$ is trivially proper since the empty set is compact. Therefore strict regularity by itself does not imply that the excess demand is invertible.

The next step is to show that under limited arbitrage the excess demand function $Z : D \longrightarrow X$ is globally invertible.

Under limited arbitrage $D \neq \emptyset$ and the image $Z(D)$ is non-empty. Furthermore, since the economy is strictly regular, by the inverse function theorem in the Appendix, the image $Z(D)$ is an open set. Next I will show that properness of $Z$ implies that the image of $Z, Z(D)$, is closed as well. Let $z = lim_{n\to\infty} z^n$ where $\forall n, z^n \in Z(D)$. The sequence $(z^n)$ is compact in $Z(D)$. As shown above, the map $Z$ is proper. By properness, $Z^{-1}\{(z^n)_{n=1,2...}\} = (p^n)_{n=1,2...}$ is a compact set, so that it contains a convergent subsequence denoted also $(p^n)$, $p^n \longrightarrow p \in Z^{-1}\{(z^n)_{n=1,2...}\}$. By continuity $Z(p) = z$, so that $Z(D)$ is a closed set. The set $D$ and its image of $Z(D)$ are non empty by limited arbitrage. Since it is both closed and open in $X$ and it is not the empty set, then $Z(D) = X$.

The next step is to show that $Z$ is a covering map for $X$. Under the hypothesis by the inverse function theorem if $Z(x) = y$ there exists neighborhoods $U_x$ and $U_y$ of $x$ and $y$ respectively, such that the restriction of the map $Z$ on $U_x$, $Z/U_x : U_x \longrightarrow U_y$, is one-to-one and onto. By the continuity of the map $Z$ , for any $y \in X$ the set $Z^{-1}(y)$ is closed; since $y$ is compact, the set $Z^{-1}(y)$ is also compact and by the inverse function theorem in the Appendix, it is 0-dimensional. Therefore for any $y \in X$, the set $Z^{-1}(y)$ consists of finitely many points. We may then choose a neighborhood $U_y$ of $y$ such that $Z^{-1}(U_y)$ consists of a union of disjoint neighborhoods each diffeomorphic to $U_y$, i.e. $Z^{-1}(U_y) = \cup_{x \in Z^{-1}(y}\{U_x\}$. This implies that $Z$ is a covering map from $D$ onto $X$.

Now I show that $Z$ is globally invertible. We know by Theorem 5 in the Appendix that for each subgroup $H$ of $\pi_1(X)$ there exists a covering $\theta : X \longrightarrow X$, which is unique up to

[12] A sequence $x^j$ is Cauchy if $\forall \varepsilon > 0 \exists N(\varepsilon) : n > N, m > M \Rightarrow \|x^n - x^m\| \leq \varepsilon$. In complete spaces, a Cauchy sequence always has a limit within a closed set; both $R^N$ and $l_2$ are complete spaces.

equivalence, such that $\theta_*(\pi_1(X)) = H$. Now observe that $H = \pi_1(X) = 0$.[13] There exists a standard map $i : D \longrightarrow X$ which defines a covering of $X$ such that $i_*(\pi_1(D)) = \pi_1(X)$. We already saw that $Z : D \longrightarrow X$ is a covering map, so that $Z_* : \pi_1(D) \rightarrow \pi_1(X)$ is a monomorphism by Theorem 3 in the Appendix. By limited arbitrage $D \neq \emptyset$ and under the conditions it is a convex set, so that its image $X$ under the excess demand function is non-empty and contractible, in particular $\pi_1(X) = 0$. Since the first homotopy group $\pi_1(X)$ is zero, $Z_*$ is onto, so that $Z_*(\pi_1(D)) = \pi_1(X)$. Therefore both maps $Z$ and $i$ satisfy $Z_*(\pi_1(D)) = i_*(\pi_1(D)) = \pi_1(X)$; it follows from Theorem 5 in the Appendix that $Z$ and $i$ define equivalent coverings. Since $i$ is a one-fold covering of $X$, then $Z$ is a globally invertible map as we wished to prove. ■

**Theorem 2** *Let $E$ be a strictly regular economy. Then $E$ has limited arbitrage if and only if it has a unique equilibrium.*

**Proof.** Sufficiency first. The global invertibility of $Z$ established in Theorem 1 above implies that there exists a unique equilibrium when the economy has limited arbitrage.

The necessity follows from Theorem 1 of [7] and its infinite dimensional counterpart in [12]: if limited arbitrage is not satisfied then a competitive equilibrium does not exist.[14] ■

# 4 Related literature on unique equilibrium

The results presented here apply only to strictly regular economies, but they encompass economies where the demand function may not be well defined at some prices, where short sales are allowed, and include finite or infinite dimensional markets. The existing literature concentrates instead on finite dimensional markets without short sales.

In finite economies without short sales the closest to Theorem 2 above is Theorem 15 on p. 236 of Arrow and Hahn [1] whose proof is connected to the convergence to equilibrium of the global Newton method. Arrow and Hahn [1] do not cover short sales, nor economies where the demand is only defined for some prices or infinite dimensional cases.

Working also on finite economies without short sales Dierker [15] assumes a desirability condition which is related but different from that required here, and uses an index argument to show the uniqueness of equilibrium. His conditions and results are different: I assume that the Jacobian never vanishes in the interior of $D$, a subset of the price space, while [15] assumes that there is a price adjustment system which is stable at each equilibrium, or more generally that the Jacobian of the system has the same sign at each equilibrium. The result obtained here is stronger than those in [15]: I prove the global invertibility of the map $Z$ and hence uniqueness of equilibrium while [15] proves that the equilibrium is unique.

The results presented here are also of a different nature from other global invertibility results for finite economies, such as the Gale-Nikaido theorem, which apply to maps defined on closed cubes and require a nonvanishing Jacobian on the interior of the cube as well as similar conditions on the boundary of the cube. I only require conditions on a convex open subset $D \subset \Delta$. Since limited arbitrage eliminates boundary equilibrium, so there is no need to study the boundary of the price space.

[13] A topological space $X$ is contractible when there exists a continuous map $F : X \times [0,1] \rightarrow X$ and $x^o \in X$ such that $\forall x \in X, F(x,0) = x$, and $F(x,1) = x^o$. A contractible space has a zero fundamental group, $\pi_1(X) = 0$. $R^N$ and $l_2$ are linear spaces and therefore contractible.

[14] For a proof that limited arbitrage is necessary and sufficient for the existence of a competitive equilibrium see also [4],[6], [7], [9].

## 5 Appendix

The following concepts and results of algebraic topology can be found in Greenberg [16]. Given two topological spaces,[15] $X$ and $Y$, $X$ is a covering space of $Y$ if there exists a continuous onto map $\theta : X \longrightarrow Y$ such that each $y \in Y$ has a neighborhood $U_y$ whose inverse image $\theta^{-1}(U_y)$ is the disjoint union of sets in $X$ each of which is homeomorphic to $U_y$. The map $\theta$ is called a covering map. When the inverse image $\theta^{-1}(y)$ of each point $y \in Y$ contains exactly $k \geq 1$ points, then the covering is called a $k-fold$ covering. The first homotopy group of $X$, also called its fundamental group, is denoted $\pi_1(X)$.Two covering spaces $p : X \longrightarrow Y$ and $p' : X' \longrightarrow Y$ are equivalent when there is a unique homeomorphism $\phi : X \longrightarrow X'$ such that $p \circ \phi = p'$.

**Theorem 3** *Let $p : X \longrightarrow Y$ be a covering map. Then $p_* : \pi_1(X) \longrightarrow \pi_1(Y)$ is one-to-one group homomorphism, i.e. a monomorphism. See [16], p. 19..*

**Theorem 4** *Any manifold $M$ has a covering space $p : X \longrightarrow M$ with $\pi_1(X) = 0$, called its 'universal' covering space. See [16], p. 23, (6.7).*

**Theorem 5** *Let $p : X \longrightarrow Y$ be a covering space. For any subgroup $H$ of $\pi_1(Y)$ there exists a covering space $p : X \longrightarrow Y$ unique up to an equivalence, such that $H = p_*\pi_1(X)$. See [16], p. 24, (6.9).*

**Theorem 6** *Inverse Function Theorem [2]. Let $M$ and $N$ be two manifolds of the same dimension, $f : M \longrightarrow N$ a smooth map, and $y = f(x)$. If the Jacobian of $f$ is non vanishing at $x$ there exists neighborhoods $U_x$ and $U_y$ of $x$ and $y$ respectively, such that $f/U_x : U_x \longrightarrow U_y$ is a diffeomorphism.[16] If $M$ is a Banach manifold then the same result holds when $f$ has an invertible Frechet derivative at $x$.*

## References


[1] Arrow, K. and F. Hahn *General Competitive Analysis*, North Holland, Amsterdam, New York and Tokyo, Fourth printing, 1986.

[2] Abraham, R. and J. Robbin *Transversal Mappings and Flows,* W.A. Benjamin, New York and Amsterdam, 1967.

[3] Chichilnisky, G. (1991) "Markets, Arbitrage and Social Choices" Paper presented at the Conference "Columbia Celebrates Arrow's Contributions" Columbia University New York October 27, 1991, CORE Discussion Paper No. 9342, CORE, Universite Catolique de Louvain, Louvain la Neuve, Belgium, 1993.

[4] Chichilnisky, Graciela (1992) "Limited Arbitrage is Necessary and Sufficient for the Existence of a Competitive Equilibrium" Columbia University Discussion Paper No. 650, December 1992.

[5] Chichilnisky, Graciela (1994) "Social Diversity and Gains from Trade: A Unified Perspective on Resource Allocation" *American Economic Review*, Papers and Proceedings, May 1994, No. 4, p. 427-434.


[15] All topological spaces are assumed to be connected and locally path connected.

[16] A diffeomorphism is a one-to-one onto map which is smooth and has a smooth inverse.

[6] Chichilnisky, G. (1994) "Limited Arbitrage is Necessary and Sufficient for the Non-emptiness of the Core", *Economic Letters*, September 1996, previously in an issue reprinted without the paper, "Limited Arbitrage is Necessary and Sufficient for the Existence of a Competitive Equilibrium and the Core and Limits Voting Cycles" *Economic Letters,* December 1994.

[7] Chichilnisky,G. (1995) "Limited Arbitrage is Necessary and Sufficient for the Existence of a Competitive Equilibrium With or Without Short Sales" *Economic Theory,* Vol. 5, No. 1, p. 79-108, January 1995.

[8] Chichilnisky, G. (1995) "A Unified Perspective on Resource Allocation: Limited Arbitrage is Necessary and Sufficient for the Existence of a Competitive Equilibrium, the Core and Social Choice" CORE Discussion Paper No. 9527.

[9] Chichilnisky, G. (1996) "A Topological Invariant for Competitive Markets" Working Paper Columbia University May 1996, to appear in *Journal of Mathematical Economics.*

[10] Chichilnisky, G. (1996) "Topology and Invertible Maps" Working Paper, Columbia University May 1996, revised June 1996.

[11] Chichilnisky, G. (1996) "Limited Arbitrage is Necessary and Sufficient for the Existence of a Competitive Equilibrium" *Journal of Mathematical Economics*, forthcoming.

[12] Chichilnisky, G. and G. Heal (1992) "Arbitrage and Equilibrium with Infinitely Many Securities and Commodities" Discussion Paper Series No. 618, Columbia University Department of Economics, July 1992, revised 1996, forthcoming in *Economic Theory.*

[13] Chichilnisky, G. and G. Heal (1984) "Existence of a Competitive Equilibrium in Sobolev Spaces without Bounds on Short Sales" *Journal of Economic Theory,* 1993, Vol. 31, No. 1, p. 68-87.

[14] Chichilnisky, G. and P. Kalman (1980) "Applcation of functional analysis to models of efficient allocation of economic resources" *Journal of Optimization Theory and Applications* 30, 19-32, 1980.

[15] Dierker, E. "Two Remarks on the Number of Equilibria of an Economy" *Econometrica,* Vol. 40, No 5, September 1972, p. 951-955.

[16] Greenberg, M. J. *Lectures on Algebraic Topology,* Mathematics Lecture Note Series, W.A. Benjamin, Inc. Reading, Massachusets, 1967.

[17] Le Van, C. "Complete characterization of Yannelis-Zame and Chichilnisky-Kalman-Mas-Colell properness conditions on preferences for separable concave functions defined on $L^p_+$ and $L^p$" *Economic Theory,* 8, No. 1, 155-176, 1996.

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